College Algebra

Chapter F Review

Note:

This review is meant to highlight basic concepts from chapter F. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

This review is available in alternate formats upon request.

A triangle is made up of 3 points: $P_1=(5,6)$ $P_2=(-2,4)$ $P_3=(1,3)$
What type of triangle is it: Isosceles (2 sides are the same), a right triangle ($a^2 + b^2 = c^2$), or neither.

Isoceles or not?

Distance Formula=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d(P_1, P_2) = \sqrt{(-2-5)^2 + (4-6)^2} = \sqrt{53}\ (\approx 7.2801)$$
$$d(P_1, P_3) = \sqrt{(1-5)^2 + (3-6)^2} = \sqrt{25} = 5$$
$$d(P_2, P_3) = \sqrt{(1-(-2))^2 + (3-4)^2} = \sqrt{10}\ (\approx 3.162)$$

None of the sides are the same length so not an isosceles triangle

Right triangle or not?

Let a=5 b=$\sqrt{10}$. C must be $\sqrt{53}$ (c is the largest number)

$$(5)^2 + (\sqrt{10})^2 = \sqrt{53}^2$$
$$25 + 10 = 53$$
$$35 = 53$$

$35 \neq 53$ So the triangle is not a right triangle.

The triangle is not an isosceles triangle nor is it a right triangle.

Find the midpoint of the line segment joining the points (-4,5) and (2,7)

Midpoint Formula =

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-4 + 2}{2}, \frac{5 + 7}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{12}{2}\right)$$

$$= (-1,6)$$

List the intercepts for the equation $y = x^3 - 27$

x-intercepts
Set y=0 and solve for x
$0 = x^3 - 27$
$27 = x^3$
$\sqrt[3]{27} = \sqrt[3]{x^3}$
$3 = x$
x-intercept is (3,0)

y-intercepts
set x=0 and solve for y
$y = (0)^3 - 27$
$y = 0 - 27$
$y = -27$
y-intercept is (0,-27)

Determine whether equation is symmetric with respect to the x-axis, y-axis, the origin or none of these.

$$y = \frac{x^4}{x^2 + 1}$$

1) Test symmetry with respect to x-axis by replacing y with –y

$$-y = \frac{x^4}{x^2+1}$$

Since $-y = \frac{x^4}{x^2+1}$ is not equivalent to $y = \frac{x^4}{x^2+1}$, the graph of the equation is not symmetric to the x-axis

2) Test symmetry with respect to y-axis by replacing x with –x

$$y = \frac{(-x)^4}{(-x)^2+1} = \frac{x^4}{x^2+1}$$

which is equivalent to $y = \frac{x^4}{x^2+1}$, so the graph of the equation is symmetric to the y-axis

3) Test symmetry with respect to the origin by replacing x with –x and y with –y

$$-y = \frac{(-x)^4}{(-x)^2+1}$$

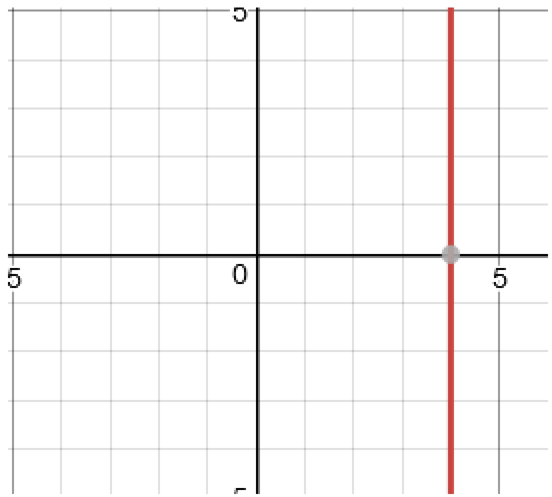
$$-y = \frac{x^4}{x^2 + 1}$$

Again since $-y = \frac{x^4}{x^2+1}$ is not equivalent to $y = \frac{x^4}{x^2+1}$, the graph of the equation is not symmetric to the origin

Find the equation of the line given that the slope is undefined and contains the point (4,9). Is the line horizontal or vertical? Graph the line.

When the slope is undefined, it means the line is a vertical line Equations of vertical lines are in the form x=x value from the point.

In this case the vertical line's equation is x=4



Find the equation of the line that contains the points (1,4) and (-1,5) . Write it in slope intercept form and general form.

1) Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{-1-1} = \frac{1}{-2} = -\frac{1}{2}$$

2) Put the slope and a point into the point-slope formula

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2}(x-1))$$

Distribute the -1/2

$$y - 4 = -\frac{1}{2}x + \frac{1}{2}$$

Add 4 to both sides

$$y = -\frac{1}{2}x + \frac{9}{2}$$

Thus the equation of the line slope-intercept point is $y = -\frac{1}{2}x + \frac{9}{2}$

3) To put in general clear the fractions and move x to the same side as y

multiple by two

$$2y = -1x + 9$$

add 1x to both sides

$$1x + 2y = 9$$

Thus the equation of the line in general form is x+2y=9

A line has the given properties: slope =2 and contains the point (3,4). Does this line also contain the points (-4,1) or (3,4)?

1) Find the equation of the line

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 3)$$

Distribute the 2

$$y - 4 = 2x - 6$$

Add four to both sides

$$y = 2x - 2$$

2) Put each point in for x and y

Point (-4,1) :

$$1 = 2(-4) - 2$$
$$1 = -8 - 2$$
$$1 = -10$$
$$1 \neq -10$$

So the line does not contain the point (-4,1)

Point(3,4):

$$4 = 2(3) - 2$$
$$4 = 6 - 2$$
$$4 = 4$$

The line contains the point (3,4)

Find the line parallel to the line y=3x+4; containing the point (1,2)

Remember parallel lines have the same slope, so the new line with have a slope of 3 just like the original line y=3x+4.

Using the fact that the slope m=3 and the line contains the point (1,2)

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 3(x - 1)$$
distribute the 3
$$y - 2 = 3x - 3$$
Add two to both sides
$$y = 3x - 1$$

Find the line perpendicular to the line y=4; containing the point (-7,8)

Remember that perpendicular lines have slopes that are negative reciprocals of each other (flip and change the sign).

For the line y=4 the slope is zero

When we flip 0 and change the sign to get $-\frac{1}{0}$ which is undefined

When the slope is undefined, it means the line is a vertical line. Equations of vertical lines are in the form x=x value from the point. In this case x=-7

So the line perpendicular to y=4 is x=-7

Find the equation of the circle in standard form where the points (0,-5) and (3,-1) are endpoints on the diameter.

1) Find the center which is the midpoint of the two given points

Midpoint =$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ =$\left(\frac{0+3}{2}, \frac{-5+(-1)}{2}\right)$ =$\left(\frac{3}{2}, \frac{-6}{2}\right)$=$\left(\frac{3}{2}, -3\right)$

Then (h,k)=$\left(\frac{3}{2}, -3\right)$

2) Find the radius. This is half the diameter (the distance between the given points).

Distance Formula=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$= $\sqrt{(3-0)^2+((-1)-(-5))^2} = \sqrt{25} = 5$

The radius is then $\frac{5}{2}$

3) Put the midpoint and the radius into the formula for a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\left(x - \frac{3}{2}\right)^2 + (y-(-3))^2 = \left(\frac{5}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 + (y+3)^2 = \frac{25}{4}$$

Find the center (h,k) and radius r of the circle. Graph the circle.

$$x^2 + y^2 - 4x + 10y - 7 = 0$$

Group the terms involving x, group the terms involving y and move the constant to the right side

$$x^2 - 4x + y^2 + 10y = 7$$

Next complete the square:

$$(x^2 - 4x + 4) + (y^2 + 10y + \_\_) = 7 + 4 + \_\_$$

$$\left(\frac{-4}{2}\right)^2 = 4$$

$$(x^2 - 4x + 4) + (y^2 + 10y + 25) = 7 + 4 + 25$$

$$\left(\frac{10}{2}\right)^2 = 25$$

$$(x^2 - 4x + 4) + (y^2 + 10y + 25) = 36$$

Factor each set of parenthesis:

$$(x-2)(x-2) + (y+5)(y+5) = 36$$
$$(x-2)^2 + (y+5)^2 = 36$$

The center (h,k)=(2,-5) and radius =6