College Algebra

Chapter 3

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 3. This review is meant to highlight basic concepts from chapter 3. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

This review is available in alternate formats upon request.

Determine if the function if is a polynomial function, a rational function, or neither. For those that are polynomial functions, state the degree. For those functions that are not polynomial functions, tell why not.

a) $f(x) = 3x^7 + 6x^2 + 15$

b) $g(x) = 3x^2 + 9x^{-1} + 7$

c) $h(x) = \frac{5x-4}{6x^2+x+3}$

a) $f(x) = 3x^7 + 6x^2 + 15$

Polynomial Function (whole number exponents)

Degree 7

b) $g(x) = 3x^2 + 9x^{-1} + 7$

neither

It has a negative exponent

c) $h(x) = \frac{5x-4}{6x^2+x+3}$

rational function because it is a $\frac{polynomial}{polynomial}$

Graph the functions using transformations (shifting, compressing, stretching, and reflection). Show all stages

$$g(x) = -(x-3)^4 + 6$$

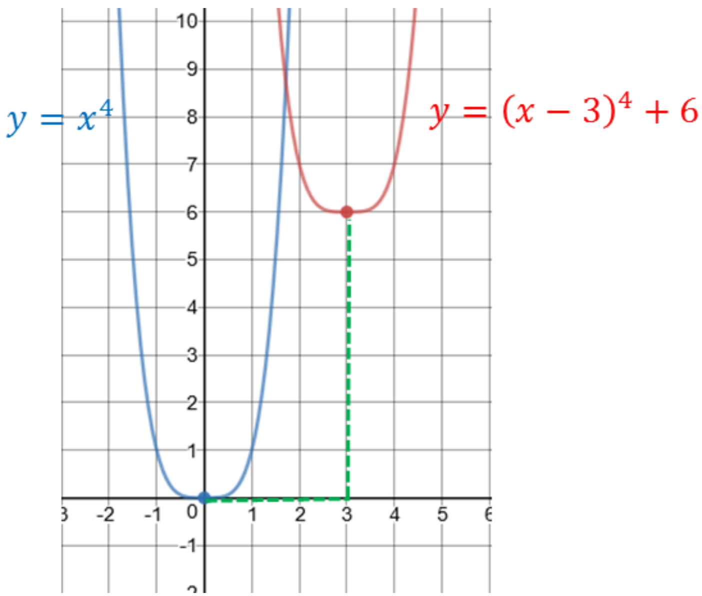
$$g(x) = -(x-3)^4 + 6$$

reflection over x-axis

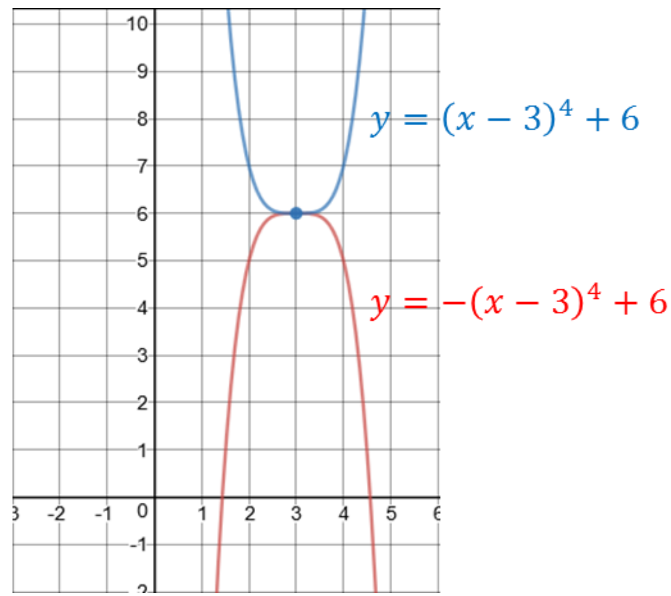
Shift 3 right

shift up 6

Shift right 3 and shift up 6



reflection over x-axis



Graph the functions using transformations (shifting, compressing, stretching, and reflection). Show all stages

$$g(x) = -(x-3)^4 + 6$$

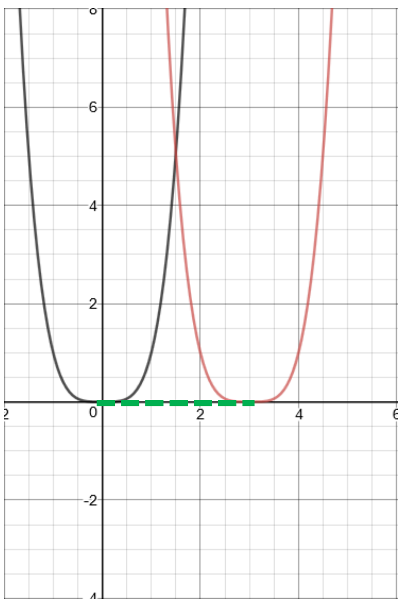
$$g(x) = -(x-3)^4 + 6$$

reflection over x-axis → $-$

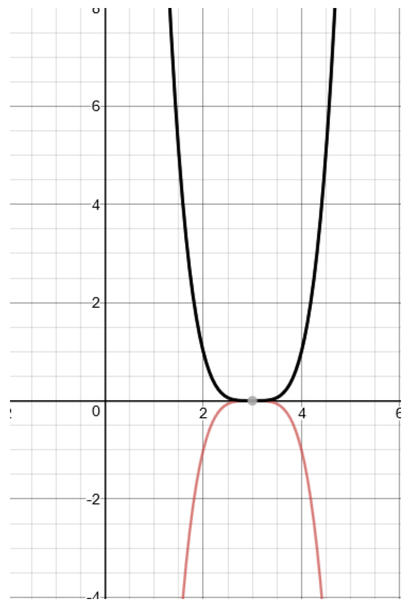
Shift 3 right → 3

shift up 6 → $+6$

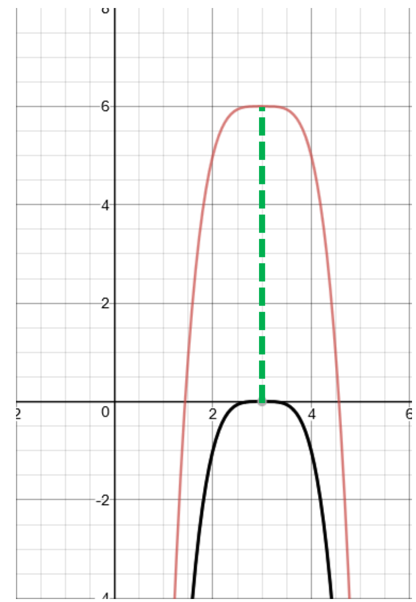
1. Shift right 3



2. Reflect over x axis



3. Shift up 6



Order for Transformations: 1. Horizontal shifts 2. Stretch/Shrink 3. Reflecting 4. Vertical Shifts

For $f(x) = -(x-1)^2(x+3)(x+1)$

a) Determine the end behavior of the graph of the function by expanding the function

b) Find the x- and y-intercepts of the graph of the function.

c) Determine the zeros of the function and their multiplicity. Does the graph cross or touch the x-axis at each x-intercept.

d) Determine the maximum number of turning points.

e) Graph the function.

a) Expanding: $f(x) = -(x-1)^2(x+3)(x+1)$

$f(x) = -(x-1)(x-1)\ (x+3)(x+1)$

$f(x) = -(x^2-2x+1)(x^2+4x+3)$

$f(x) = -(x^2-2x+1)\ (x^2+4x+3)$

$f(x) = -(x^4+2x^3-4x^2-2x+3)$

$f(x) = -x^4-2x^3+4x^2+2x-3$

The degree of the function is 4 so both ends either rise or both fall and a<0, so the function is falling at both ends.

b) x-intercepts: set y=0 and solve for x

$$0 = -(x-1)^2(x+3)(x+1)$$

0=x-1	0=x+2	0=x+1
x=1	x=-3	x=-1

x-intercepts are 1,-3,-1

y-intercepts: set x=0 and solve for y

$$f(0) = -(0-1)^2(0+3)(0+1)$$

$$f(0) = -(-1)^2(3)(1) = -3$$

y-intercept is -3.

Continued on next slide…

For the following polynomial

a) Determine the end behavior of the graph of the function by expanding the function

b) Find the x- and y-intercepts of the graph of the function.

c) Determine the zeros of the function and their multiplicity. Does the graph cross or touch the x-axis at each x-intercept.

d) Determine the maximum number of turning points.

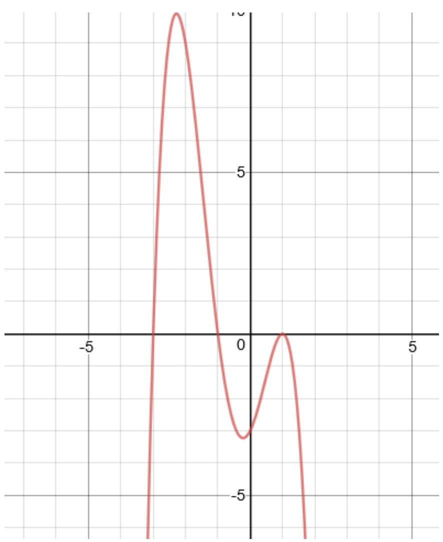
e) Graph the function.

$$f(x) = -(x-1)^2(x+3)(x+1)$$

c) 1 has multiplicity of 2. The graph touches the x-axis at x=1.
-3 has multiply of 1. The graph crosses the x-axis at x=-3.
-1 has multiplicity of 1. The graph crosses the x-axis at x=-1.

d) The number of turning points =degree -1
4-1=3 turning points

e) Graph:



Find the remainder R when f(x) is divided by g(x). Is g a factor of f?

$$f(x) = x^3 - 4x^2 + 10x - 4$$
$$g(x) = x - 2$$

Using the remainder theorem.
$g(x) = x - 2$ So x=2

$$f(2) = (2)^3 - 4(2)^2 + 10(2) - 4 = 8$$

The remainder R when f(x) is divided by g(x) is 8.
Since the remainder is not zero g is not a factor of f.

Use Descartes' Rule of Signs to determine how many positive and negative zeros the polynomial function may have. Do not find the zeros.

$$g(x) = 15x^9 - 6x^7 - 3x^5 + 4x^2 - 6$$

Positive Zeros

Count how many changes in signs there are. Keep subtracting 2 from zero until its no longer possible (3-2)=1

So there are 3 or 1 positive zero

$$g(x) = 15x^9 \quad - 6x^7 \quad - 3x^5 \quad + 4x^2 \quad - 6$$

+ to - - to + + to -

Negative Zeros

Substitute x with –x and simplify

$$g(-x) = 15(-x)^9 - 6(-x)^7 - 3(-x)^5 + 4(-x)^2 - 6$$

$$g(-x) = -15x^9 + 6x^7 + 3x^5 + 4x^2 - 6$$

Count how many changes in signs there are. Keep subtracting 2 from zero until its no longer possible (2-2)=0

So there are 2 or 0 negative zeros

$$g(-x) = -15x^9 + 6x^7 + 3x^5 + 4x^2 - 6$$

- to + + to -

Use the Rational Zeros Theorem to find all the rational zeros of the polynomial function. Use the zeros to factor f over the real numbers.

$$h(x) = 2x^3 + 3x^2 - 11x - 6$$

For p (using 6), the factors are $\pm 1, \pm 2 \pm 3 \pm 6$

For q (using 2), the factors are $\pm 1, \pm 2$

Then $\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

Substitute each $\frac{p}{q}$ in for x (for example $h(1) = 2(1)^3 + 3(1)^2 - 11(1) - 6 = -12$

Zeros indicate rational zeros for the polynomial.

Potential zeros	*Value of* $h(\frac{p}{q})$	*Potential zeros*	*Value of* $h(\frac{p}{q})$	*Potential zeros*	*Value of* $h(\frac{p}{q})$
1	-12	3	42	$\frac{1}{2}$	-10.5
-1	6	-3	0	$-\frac{1}{2}$	0
2	0	6	-264	$\frac{3}{2}$	-9
-2	12	-6	468	$-\frac{3}{2}$	10.5

The rational zeros are 2, -3, $-\frac{1}{2}$. Using this and the leading coefficient 2, h(x) in factored form

$h(x) = 2(x - 2)(x + 3)(x + \frac{1}{2})$

Find bounds to the real zeros of each polynomial function

$$f(x) = x^3 - 3x^2 + 4x - 12$$

Using the rational zero test, potential rational zeros are: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

To find an upper bound take the smallest positive integer that is also possible rational zero (1)

Use this with synthetic division

1	1	-3	4	-12
	↓	1	-2	2
	1	-2	2	-10

The value 1 is not an upper bound for the real zeros of f(x) because the last row of synthetic division contains a negative value.

We keep repeating this for every positive x-values (not shown). If all have at least one negative coefficient, then use the next integer and so until no coefficient is negative (in this case it would be 13,14,15…).

The table below shows the last row from synthetic division

Possible zeros	*Coefficients from the last row of synthetic division*			*remainder*
1	1	-2	2	-10
2	1	-1	2	-8
3	1	0	1	0

← We can stop here as this is the first line without any negative coefficients. 3 is the upper bound.

Continued on next slide...

Find bounds to the real zeros of each polynomial function

$$f(x) = x^3 - 3x^2 + 4x - 12$$

To find the lower bound take the negative integer closest to zero that is also possible rational zero (-1)

Use this with synthetic division

-1	1	-3	4	-12
	↓	-1	4	-8
	1	-4	8	-20

The value -1 is the lower bound for the real zeros of f(x) because in the last row of synthetic division contains coefficients that are alternating in signs.

If this was not the case, we would have kept repeating this for every negative x-values (not shown). If all have do not have alternating signs, then use the next integer and so until the there is coefficients with alternating signs. (In this case it would be -13,-14,-15…)

The lower bound is -1 and the upper bound is 3.

Use the Intermediate Value Theorem to show that polynomial has a zero in the given interval

$$g(x) = 3x^3 - x - 1; [0,1]$$

$g(0) = 3(0)^3 - (0) - 1 = -1$
$g(1) = 3(1)^3 - (1) - 1 = 1$

The value of the function is positive at one endpoint (x=0) and negative at the other (x=1). Since the function is continuous, the function must cross the x-axis at some point for the y-values to go from negative to positive. Thus the intermediate value theorem guarantees at least one zero in the interval [0,1].

Using the given information to find the other zeros and to find a polynomial function with real coefficients that has the zeros.
Degree 3; zeros $2 + i$, 5

Degree 3 means three zeros. Remember complex zeros come in conjugate pairs.
Zeros: 5, $2 + i$, $2 - i$,

$$f(x) = (x - 5)(x - 2 + i)(x - 2 - i)$$
FOIL $(x - 2 + i)(x - 2 - i)$
$$f(x) = (x - 5)(x^2 - 2x - ix - 2x + 4 + 2i + ix - 2i - i^2)$$
Combine like terms
$$f(x) = (x - 5)(x^2 - 4x + 4 - i^2)$$
Remember $i^2 = -1$
$$f(x) = (x - 5)(x^2 - 4x + 4 - (-1))$$
Combine like terms
$$f(x) = (x - 5)(x^2 - 4x + 5)$$
FOIL
$$f(x) = x^3 - 4x^2 + 5x - 5x^2 + 20x - 25$$
Combine like terms
$$f(x) = x^3 - 9x^2 + 25x - 25$$

$f(x) = x^3 - 9x^2 + 25x - 25$ is a polynomial function that satisfies the following conditions: degree 3; zeros $2 + i$, 5

Solve the equation $x^4 + x^3 - 3x^2 + 3x - 18 = 0$

1) The degree is 4, so that is the maximum number of zeros.

2) Use Descartes rule of signs
positive: $x^4 + x^3 - 3x^2 + 3x - 18$
3 or 1 positive real zeros
Negative zeros (using –x)
$(-x)^4 + (-x)^3 - 3(-x)^2 + 3(-x) - 18$
$x^4 - x^3 - 3x^2 - 3x - 18$
There is only one negative real zero

3) Use Rational Zero test
p: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
q: ± 1,
Then $\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
When we substitute $\frac{p}{q}$ for x, the only values that are zero are -3 and 2…

These are the only rational real zeros but there will be irrational zeros/complex zeros. So we use synthetic division to diminish the function.

-3	1	1	-3	3	-18
	↓	-3	6	-9	18
	1	-2	3	-6	0

$x^3 - 2x^2 + 3x - 6$

2	1	-2	3	-6
	↓	2	0	6
	1	0	3	0

$x^2 + 3$

4) Solve the diminished function

$$x^2 + 3 = 0$$
$$x^2 = -3$$
$$x = \pm\sqrt{-3}$$
$$x = \pm i\sqrt{3}$$

The zeros are -3, 2, $\pm i\sqrt{3}$

Find the domain of each rational function. Find any vertical, horizontal, or oblique asymptotes.

$$f(x) = \frac{5x^2}{x^2+9}$$

Domain: Set the denominator to zero and solve for x. x cannot be these numbers

$$x^2 + 9 = 0$$
$$x^2 = -9$$
$$x = \pm 3i$$

Since the domain only deals with real numbers not imaginary, it doesn't matter that $x \neq \pm 3i$. The domain is all real numbers.

Vertical: Set the denominator to zero and solve for x; these are the asymptotes. Since vertical asymptotes only deal with real numbers not imaginary, it doesn't matter that $x \neq \pm 3i$. There is no vertical asymptote.

Horizontal: The degree of the numerator and the denominator are the same, we divide the leading coefficients.

$$y = \frac{5}{1} = 5$$

y=5 is the horizontal asymptote.

Oblique: When there is a horizontal asymptote, there is not an oblique asymptote.

$$g(x) = \frac{x^2+5x-3}{x+1}$$

Domain: Set the denominator to zero and solve for x. x cannot be these numbers

$$x + 1 = 0$$
$$x = -1$$

The domain is $\{x | x \neq -1\}$

Vertical: Set the denominator to zero and solve for x, these are the asymptotes. See Domain
vertical asymptote: $x = -1$

Horizontal: Since the degree of the numerator is one more than the degree of the denominator, there is not a horizontal asymptote, but there is an oblique asymptote.

Oblique: Divide the numerator by the denominator. Using synthetic (long division is ok too) and ignore any remainder

Oblique asymptote: y=x+4

-1	1	5	-3
	↓	-1	-4
	1	4	-7

Analyze the rational function graph the rational function. That is graph the function by hand (see steps on page 246).

$$F(x) = \frac{x^2+7x+10}{x^2-x-6}$$

1) Factor and find the domain.

$$F(x) = \frac{x^2+7x+10}{x^2+4x+2} = \frac{(x+2)(x+5)}{(x+2)(x-3)}$$

Domain: $(x+2)(x-3) = 0$

$$x \neq -2, 3$$

2) Lowest terms

$$F(x) = \frac{(x+2)(x+5)}{(x+2)(x-3)} = \frac{x+5}{x-3}$$

There is a hole at x=-2.

3) Intercepts.

y-intercept: $F(0) = \frac{0+5}{0-3} = -\frac{5}{3}$

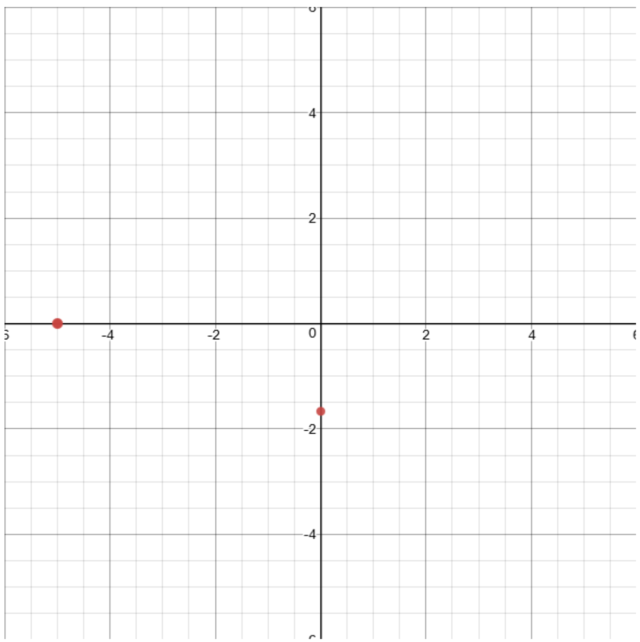
x-intercept: $0 = \frac{x+5}{x-3}$ (multiply both sides by x-3)

$$0 = x + 5$$

x=-5

Since the multiplicity is 1, the function crosses the x-axis at x=-5.

Plot these intercepts.



Analyze the rational function graph the rational function. That is graph the function by hand (see steps on page 246). $F(x) = \frac{x^2+7x+10}{x^2-x-6}$

4) Vertical asymptotes

$$F(x) = \frac{x+5}{x-3}$$
$$x - 3 = 0$$
$$x = 3$$

Vertical Asymptote x=3
Plot this as dotted line.

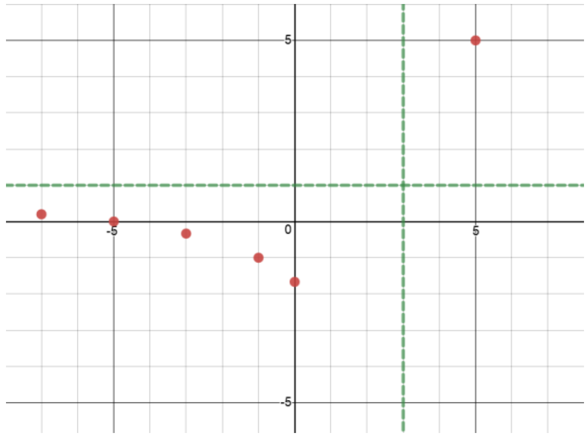
5) Horizontal/Oblique Asymptotes
The degrees are the same, so the horizontal asymptotes is $y = \frac{1}{1} = 1$. Plot this as dotted line

6) Is the function lies above/below the x-axis? Plot these points.

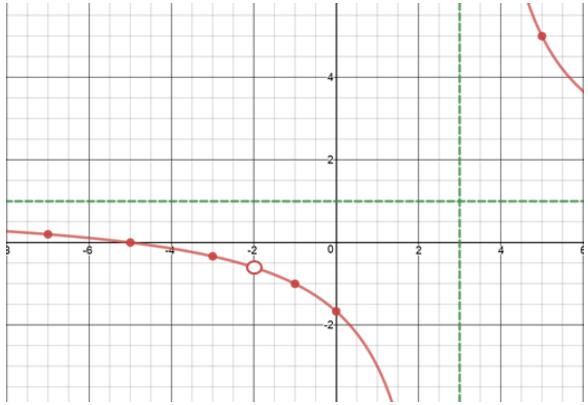
Interval	$(-\infty$,-5)	(-5,-2)	(-2,3)	$(3,\infty)$
Number chosen	-7	-3	-1	5
Value of F	.2	-.33	-1	5
Location	above	below	below	above
point	(-7,.2)	(-3,-.33)	(-1,-1)	(5,5)

7) Graph the function

Steps 4-6



Steps 7



Solve the inequality and graph the solution set.

$$\frac{(x+3)(x-2)}{(x-4)} < 0$$

The zero's are

$(x+3)(x-2)=0$

$x=-3,2$

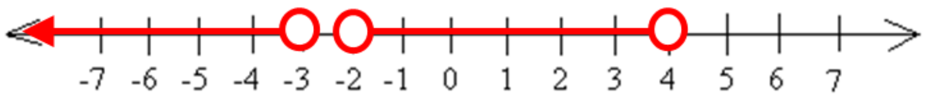
The function is undefined at x=4

Interval	$(-\infty,-3)$	$(-3,2)$	$(2,4)$	$(4,\infty)$
Number chosen	-4	0	3	5
Values of f	$-\frac{3}{4}$	$\frac{3}{2}$	-6	24
Conclusion	negative	positive	negative	positive

When looking for <0, look for where the conclusion is negative.

The solution set is: $(\infty,-3)\cup(2,4)$

The Graph of the solution set is



The concentration C (measured in mg) of a certain drug in a patient's bloodstream after t hours after injection is given by

$$C(t) = \frac{7t+9}{t^2+1}$$

a) When will the concentration be 1 mg

b) When will the drug be at its highest concentration.

a) $1 = \frac{7t+9}{t^2+1}$

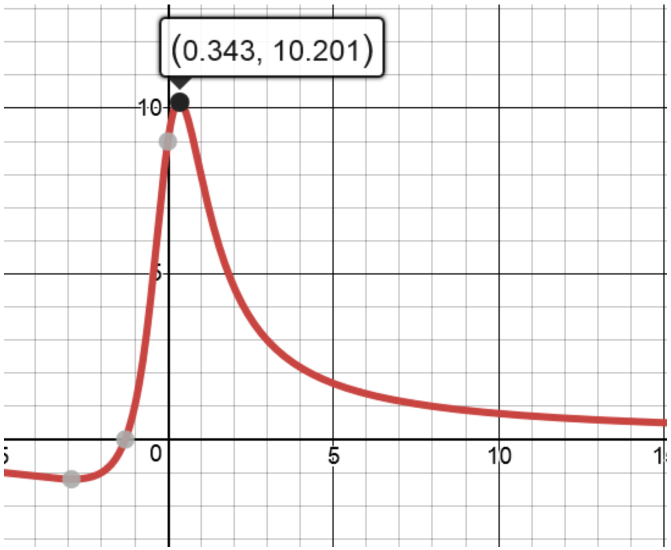
$$t^2 + 1 = 7t + 9$$
$$t^2 - 7t - 8 = 0$$

Solve this by either factoring, using the quadratic formula or completing the square.

$$(t-8)(t+1) = 0$$
$$t = 8, -1$$

Can't have negative time, so the concentration is 1 after 8 hours.

b) Graph the function to find the maximum



The drug will be at the highest concentration at .343 hours.