College Algebra

Chapter 6

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 6. This review is meant to highlight basic concepts from chapter 6. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

Solve the system of equations using the method of substitution or elimination.

$$2x + 4y = 10$$
$$4x + 3y = -30$$

Subsitution

$2x + 4y = 10$ (eq 1)
$4x + 3y = -30$ (eq 2)

Solve the first equation for x.

$$2x + 4y = 10$$
$$2x = 10 - 4y$$
$$x = 5 - 2y \text{ (eq 3)}$$

Substitute x into the second equation and solve for y.

$$4(5 - 2y) + 3y = -30$$
$$20 - 8y + 3y = -30$$
$$20 - 5y = -30$$
$$-5y = -50$$
$$y = 10$$

Substitute y into equation 3 to solve for x.

$$x = 5 - 2(10)$$
$$x = -15$$

Solution is $(-15,10)$

Elimination

$2x + 4y = 10$ (eq 1)
$4x + 3y = -30$ (eq 2)

Multiply the first equation by negative two and

$$-2(2x + 4y = 10)$$
$$-4x - 8y = -20$$

Add this to equation 2.

$$\begin{array}{r} -4x - 8y = -20 \\ 4x + 3y = -30 \\ \hline -5y = -50 \end{array}$$

Solve for y

$$-5y = -50$$
$$y = 10$$

Substitute y into equation either equation and solve for x.

$$2x + 4(10) = 10$$
$$2x = -30$$
$$x = -15$$

Solution is $(-15,10)$

Solve the system of equations.

$$x - y + 2z = 4$$
$$2x + 2y + z = -4$$
$$-x + 3y + 2z = -10$$

This is one way to solve using elimination.

$$x - y + 2z = 4 \quad (eq\ 1)$$
$$2x + 2y + z = -4 \quad (eq\ 2)$$
$$-x + 3y + 2z = -10 \quad (eq\ 3)$$

Add equations 1 and 2 to eliminate x

$$x - y + 2z = 4$$
$$-x + 3y + 2z = -10$$
$$\overline{\qquad 2y + 4z = -6 \quad (eq\ 4)}$$

Multiply equation 3 by 2 and add to equation 2 in order to eliminate the x again.

$$2(-x + 3y + 2z = -10)$$
$$-2x + 6y + 4z = -20$$

$$-2x + 6y + 4z = -20$$
$$2x + 2y + z = -4$$

$$8y + 5z = -24 \quad (eq\ 5)$$

Use equations 4 and 5

$$2y + 4z = -6 \quad (eq\ 4)$$
$$8y + 5z = -24 \quad (eq\ 5)$$

Multiply equation 4 by -4

$$-4(2y + 4z = -6)$$
$$-8y - 16z = 24$$

Add this to equation 5 to eliminate the y.

$$-8y - 16z = 24$$
$$8y + 5z = -24$$
$$\overline{\qquad -11z = 0 \qquad}$$

Solving for z, we get $z = 0$.

Substitute the known value of z into equations 4 or 5 and solve for y.

$$2y + 4(0) = -6$$
$$2y = -6$$
$$y = -3$$

Substitute the known values of y and z into any of the three original equations and solve for x.

$$x - (-3) + 2(0) = 4$$
$$x + 3 = 4$$
$$x = 1$$

The solution set is $(1, -3, 0)$

Use the following matrices to compute AB and BA

$$A = \begin{bmatrix} -1 & -2 \\ 2 & -5 \\ 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 1 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Make sure we can multiply
A is a 3 by 2 and B is a 2 by 3

AB

A — 3 by 2
B — 2 by 3

Same (inner numbers)
dimensions of AB (outer numbers)

$$\begin{bmatrix} -1\cdot 6 + (-2)\cdot 2 & -1\cdot 1 + (-2)\cdot 4 & -1\cdot 3 + (-2)\cdot 5 \\ 2\cdot 6 + (-5)\cdot 2 & 2\cdot 1 + (-5)\cdot 4 & 2\cdot 3 + (-5)\cdot 5 \\ 3\cdot 6 + 0\cdot 2 & 3\cdot 1 + 0\cdot 4 & 3\cdot 3 + 0\cdot 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -9 & -13 \\ 2 & -18 & -19 \\ 18 & 3 & 9 \end{bmatrix}$$

B — 2 by 3
A — 3 by 2

Same (inner numbers)
dimensions of BA (outer numbers)

$$\begin{bmatrix} 6\cdot -1 + 1\cdot 2 + 3\cdot 3 & 6\cdot -2 + 1\cdot -5 + 3\cdot 0 \\ 2\cdot -1 + 4\cdot 2 + 5\cdot 3 & 2\cdot -2 + 4\cdot -5 + 6\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -17 \\ 21 & -24 \end{bmatrix}$$

Find the Inverse of the Matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$$

1. Form the Matrix $[\mathrm{A}|I_n]$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$$

2. Transform the matrix $[\mathrm{A}|I_n]$ into reduced echelon form

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] R_2 = -2r_1 + r_2 \rightarrow \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] R_2 = -1r_2 \rightarrow \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$$R_1 = r_1 - 4r_2 \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -7 & 4 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

3. The echelon form of the matrix $[\mathrm{A}|I_n]$ contains the inverse

The inverse is $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

Solve the system using matrices

$$2x + y + z = 5$$
$$x - 1y - 3z = 4$$
$$8x + y - 1z = 5$$

(Note different steps will get you the same answer)

Corresponding Augmented Matrix

$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 1 & -1 & -3 & 4 \\ 8 & 1 & -1 & 5 \end{array}\right]$ Switch R_1 and R_2. $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 2 & 1 & 1 & 5 \\ 8 & 1 & -1 & 5 \end{array}\right]$ $2R_1 - R_2$ $into$ R_2 $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 0 & -3 & -7 & 3 \\ 8 & 1 & -1 & 5 \end{array}\right]$

$-8R_1 + R_3$ $into$ R_3 $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 0 & -3 & -7 & 3 \\ 0 & 9 & 23 & -27 \end{array}\right]$ $-\frac{1}{3}R_2$ $into$ R_2 $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 0 & 1 & \frac{7}{3} & -1 \\ 0 & 9 & 23 & -27 \end{array}\right]$

$-9R_2 + R_3$ $into$ R_3 $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 0 & 1 & \frac{7}{3} & -1 \\ 0 & 0 & 2 & -18 \end{array}\right]$ $\frac{1}{2}R_3$ $into$ R_3 $\left[\begin{array}{ccc|c} 1 & -1 & -3 & 4 \\ 0 & 1 & \frac{7}{3} & -1 \\ 0 & 0 & 1 & -9 \end{array}\right]$

We could continue to get the matrix in reduced row echelon form but we are going to use backward substitution instead.

From row 3: z=-9

From row 2: $y + \frac{7}{3}z = -1$ $\quad y + \frac{7}{3}(-9) = -1$ solving for y we get y=20

From row 1: $x - y - 3z = 4$ $\quad x - 20 - 3(-9) = 4$ solving for x we get x=-3

Solution is (-3,20,-9)

Find the Determinant

$$\begin{vmatrix} 2 & 1 & -3 \\ 5 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix}$$

$$\det\begin{vmatrix} 2 & 1 & -3 \\ 5 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix} = 2 \cdot det \begin{bmatrix} 0 & 1 \\ 6 & 0 \end{bmatrix} - 1 \cdot det \begin{vmatrix} 5 & 1 \\ 2 & 0 \end{vmatrix} + (-3) \cdot det \begin{vmatrix} 5 & 0 \\ 2 & 6 \end{vmatrix}$$

$$= 2 \cdot (0 \cdot 0 - 6 \cdot 1) - 1 \cdot (5 \cdot 0 - 1 \cdot 2) + (-3) \cdot (5 \cdot 6 - 2 \cdot 0)$$

$$= 2 \cdot (-6) - 1 \cdot (-2) + (-3) \cdot (30)$$

$$= -100$$

Solve using Cramer's Rule

$$3x - 3y = 5$$
$$4x - 5y = 6$$

Cramer's rule

For $\begin{matrix} ax + by = s \\ cx + dy = t \end{matrix}$ $x = \frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ $and\ y = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$$x = \frac{\begin{vmatrix} 5 & -3 \\ 6 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -3 \\ 4 & -5 \end{vmatrix}} = \frac{(5(-5)-3(-6))}{(3(-5)-4(-3))} = \frac{-7}{-3} = \frac{7}{3}$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -3 \\ 4 & -5 \end{vmatrix}} = \frac{(3(6)-4(5))}{(3(-5)-4(-3))} = \frac{-2}{-3} = \frac{2}{3}$$

Solution is $(\frac{7}{3}, \frac{2}{3})$

Write the partial fraction decomposition for the rational expression

$$\frac{x}{(x^2+9)(x+1)}$$

$$\frac{x}{(x^2+9)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

Multiply both sides by the LCD $(x^2+9)(x+1)$

$x = A(x^2+9) + (\mathrm{B}x + \mathrm{C})(x+1)$

Let x=-1. Then

$-1 = A\left((-1)^2+9\right) + (\mathrm{B}(-1)+\mathrm{C})((-1)+1)$

$-1 = A(10) + (\mathrm{B}(-1)+\mathrm{C})(0)$

$-1 = A(10)$

$A = -\frac{1}{10}$

Let x=0. Then

$0 = A\left((0)^2+9\right) + (\mathrm{B}(0)+\mathrm{C})((0)+1)$

$0 = A(9) + (\mathrm{C})(1)$

$0 = (-\frac{1}{10})(9) + (\mathrm{C})(1)$

$C = \frac{9}{10}$

Let x=1

$1 = A\left((1)^2+9\right) + (\mathrm{B}(1)+\mathrm{C})((1)+1)$

$1 = A(10) + (\mathrm{B}+\mathrm{C})(2)$

$1 = 10A + 2\mathrm{B} + 2\mathrm{C}$

$1 = 10(-\frac{1}{10}) + 2\mathrm{B} + 2(\frac{9}{10})$

$1 = -1 + 2\mathrm{B} + (\frac{9}{5})$

$B = \frac{1}{10}$

$$\frac{x}{(x^2+9)(x+1)} = \frac{-\frac{1}{10}}{x+1} + \frac{\frac{1}{10}x+\frac{9}{10}}{x^2+9}$$

$$= \frac{-1}{10(x+1)} + \frac{1x+9}{10(x^2+9)}$$

Solve the system of inequalities

$$x^2 + y^2 = 6$$
$$x^2 + 2y = 6$$

Solving by substitution

Solve the second equation for x^2

$$x^2 + 2y = 6$$
$$x^2 = -2y + 6$$

Substitute into the first equation

$$x^2 + y^2 = 6$$
$$(-2y + 6) + y^2 = 6$$

Bring everything to the left hand side

$$y^2 - 2y - 6 + 6 = 0$$
$$y^2 - 2y = 0$$

Solve for y using factoring

$$y(y - 2) = 0$$
$$y = 0 \; and \; y = 2$$

Substitute these values into y and solve for x

y=0: $x^2 + 2(0) = 6$

$$x = \pm\sqrt{6}$$

y=2: $x^2 + 2(2) = 6$

$$x = \pm\sqrt{2}$$

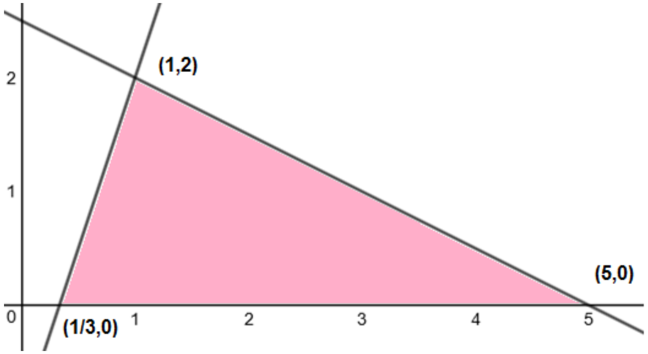
Solutions are $(\sqrt{2}, 2)$, $(-\sqrt{2}, 2)$, $(\sqrt{6}, 0)$, $(-\sqrt{6}, 0)$

*Problem can be solved by elimination

Solve

Maximize $C = 4x + 7y$ subject to $x \geq 0, y \geq 0, x + 2y \leq 5, 3x - y \geq 1$

Graph the inequalities



The Vertices are $(1,2), (5,0), and\ \left(\frac{1}{3}, 0\right)$

Evaluating the Objective function

Vertex	Value of $C = 4x + 7y$
$(1,2)$	$C = 4(1) + 7(2) = 18$
$(5,0)$	$C = 4(5) + 7(0) = 20$
$\left(\frac{1}{3}, 0\right)$	$C = 4\left(\frac{1}{3}\right) + 7(0) = \frac{4}{3}$

The maximum is 20 at (5,0).

All the rooms in a house need to be cleaned.
If Cody and Rose working together will finish the job in 1 hour 30 minutes. If Rose and Paige work together, the job will be completed in 1 hour and 15 minutes. If Cody and Paige work together, the house will be cleaned in 45 minutes. How long will it take each of them working alone to finish the job?

Put all the time into minutes. Let c=Cody, r=Rose and p=Paige
Cody and Rose finish in 1 hour 30 minutes: c+r=90
Rose and Paige finish in 1 hour 15 minutes: r+p=75
Cody and Paige finish in 45 minutes: c+p=45
System of equations is

$$c + r + 0p = 90$$
$$0c + r + p = 75$$
$$c + 0r + p = 45$$

Solving the system using any method
c=30, r=60, p=15
Cody will take 30 minutes, Rose will take 60 minutes, and Paige will take 15 minutes.