Finding the Equation of a Line

Lines can come in three forms:

- Ax+By=C where A, B and C are numbers and the slope is $-\frac{A}{B}$
- Slope-intercept form: y=mx+b where m is the slope and the b is the y-intercept
- Point-slope form: $y-y_1=m(x-x_1)$ or $y= m(x-x_1)+ y_1$ where m is the slope and x_1 and y_1 are from a given point

Given Slope and the Y-Intercept

When we are given the slope, it is labeled slope= or m=. The y-intercept is given as (0,b). We take this information and plug it into the slope-intercept form.

Example: Find the equation of the line where m=9 and goes through the point (0,6).

Here are slope is 9 (m) and the y-intercept is 6 (b). Putting this information in the slope-intercept form, we get y=9x+6.

Given the Slope and a Point

Sometimes we are given the slope and a point that is not the y-intercept. If we want the answer is point-slope form, we put the slope in for m and the point into x_1 and y_1 into the form. If we want the equation in point-intercept form, we have 2 options. Both get the same answer in the end. The method you choose is a matter of preference.

A) Point-Slope Method:
   1. Put the information into the point-slope form
   2. Distribute the m
   3. Add/Subtract y_1 to both sides (using addition or subtraction)
   4. Combine like terms to get y=mx+b form

B) Slope-Intercept Method:
   1. Temporarily put the the slope in for m and the x and y values from the point into x and y.
   2. Mulitply the m and the x value
   3. Solve for b by adding/subtracting the number from step 2 to both sides
   4. Combine like terms to get b
   5. Put the slope and b into y=mx+b. Leave y as y and x as x (not as the values from the point)

Example: Find the equation of the line (in point-slope form) that has a slope of 3 and goes through the point (1,4).

Here our m=3, our x_1=1 and y_1=4. Putting the information into the formula we get

y-4=3(x-1) or *y=3(x-1)+4*

Example: Find the equation of the line (in slope-intercept form) that has a slope of 3 and goes through the point (1,4).

Point-Slope Method	Slope-Intercept Method
First, we put the information into point-slope form. $y-y_1=m(x-x_1)$ *y-4=3(x-1)* Distribute the 3. *y-4=3x-3* Add four to both sides *y-4+4=3x-3+4* *y=3x-3+4* Combine like terms *y=3x+1* Our equation is now in slope-intercept form.	First, we put the slope in for m and we *temporarily* put our x and y from our point into the slope-intercept form. *y=mx+b* *4=3(1)+b* Multiply the 3 and the 1 *4=3+b* Solve for b subtracting both sides and combining like terms *4-3=3-3+b* *1=b.* Put the m and b into the form *y=3x+1.* Notice that once we found b, x goes back to being x and y goes back to being y.

Given Two Points

Sometimes we are given two points (x_1,y_1) and (x_2,y_2) and we need to find the line. First, we need find the slope. The formula for slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Once we know the slope, we either

a) Put the slope and one of the points (doesn't matter which one) into the point-slope form
b) With the slope and either of the points, use either the point-slope method or the slope-intercept method to get the equation in y=mx+b form.

Example: Find the equation of the line (in slope-intercept form) that goes through the points (1,5) and (2,7).

First, we find the slope. Here (x_1,y_1)= (1,5) and (x_2,y_2)= (2,7). Putting this information into the slope intercept form, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{2-1} = \frac{2}{1} = 2$$

Now we can use either method to find the equation.

Point-Slope Method	Slope-Intercept Method
We will put the slope in for m and the first point (1,5) into the point-slope form. *y-5=2(x-1)* Distribute the 2 *y-5=2x-2* We add 5 to both sides *y-5+5=2x-2+5* *y=2x-2+5* Combine like terms *y=2x+3* We have our equation!	We will put the slope in for m and the first point (1,5) into the slope-intercept form. *5=2(1)+b* Multiply the 2 and the 1 *5=2+b* Subtract 2 to both sides *3=b* Putting our slope and b into y=mx+b we get *y=2x+3* We have our equation!

Find a Line Parallel to a Line that Goes Through a Given Point

REMEMBER: Two lines are parallel if the slope is the same.

A commonly seen problem is that given an equation and a point (x,y), you are asked to find a line parallel to the given equation and that your parallel line goes through the point (x,y). To solve this type of problem,

1) Identify the slope of the equation given. Ignore the y-intercept.
2) Using the slope and the point given, find the equation of the line using either method described above.

Example: Find a line parallel to y=3x+2 that goes through the point (1,4).

1) We identify the slope. Since our equation is given in y=mx+b, our slope is 3.
2) Now that we have the slope for our new line, we will use the point (1,4) to write the equation using the point-slope method or the slope-intercept method

Point-Slope Method		Slope-Intercept Method	
$y-y_1=m(x-x_1)$		Y=mx+b	
y-4=3(x-1)	Put in the information	4=3(1)+b	Temporarily put in the information
y-4=3x-3	Distribute the 3	4=3+b	Multiply 3 and 1
y=3x-3+4	Add 4 to both sides	1=b	Subtract 3 from both sides
y=3x+1	Add like terms	Y=3x+1	Put in the important information

Find a Line Perpendicular to a Line that Goes Through a Given Point

REMEMBER: Two lines are perpendicular if the slopes are negative reciprocals of each other.

As with parallel lines, a commonly seen problem is that given an equation a point (x,y), you are asked to find a line perpendicular to the given equation and that your parallel line goes through the point (x,y). To solve this type of problem,

1) Identify the slope of the equation given.
2) Find the negative reciprocal (i.e. flip the slope and change the sign). This is the slope for the perpendicular line.
3) Using the slope and the point given, find the equation of the line using either method described above.

Example: Find a line perpendicular to y=-3x+2 and goes through the point (3,-2).

1) The Slope of the given equation is -3. We know this because our equation is given in y=mx+b form and the -3 is in the m position.
2) The negative reciprocal of -3 is $\frac{1}{3}$, which is the slope of our perpendicular line. This is because
   a. -3 can be written as $-\frac{3}{1}$
   b. Flipping $-\frac{3}{1}$ we get $-\frac{1}{3}$
   c. Changing the sign to a positive we get $\frac{1}{3}$
3) Now using the slope of 1/3 and the point (3,-2), we can find the equation of the line.

Point-Slope Method	Slope-Intercept Method
$y-y_1=m(x-x_1)$	$y=mx+b$
$y-(-2)=\frac{1}{3}(x-3)$	$-2=\frac{1}{3}(3)+b$
$y+2=\frac{1}{3}(x-3)$	$-2=1+b$
$y+2=\frac{1}{3}x-1$	$-3=b$
$y=\frac{1}{3}x-1-2$	$y=\frac{1}{3}x-3$
$y=\frac{1}{3}x-3$	

Vertical Lines

A vertical line is given as x=k, where k is a real number. The slope for a vertical line is undefined. There are four main types of questions concerning finding vertical lines.

1) If you are asked to find a vertical line given an undefined slope and a point, the vertical line is x= x-value of the point.

   Example: Find the equation of the line given m= undefined and (3,2).
   Since slope is undefined, we have a vertical line. The equation of the line is x=3 (3 is the x-value of the point).

2) If you are given two points and the x-values for the two points are the same, then the equation of the line that goes through those points is vertical. The line is written x= x-value from the points. If you use the slope formula, you would get a zero in the denominator, which means the slope is undefined.

   Example: Find the equation of the line that goes through the points (-4,-9) and (-4,14).
   Notice that the x-values are the same for both points. This means we have a vertical line x=-4 (-4 is the x-values for both points).

3) If you are asked to find a line parallel given a vertical line and a point, your parallel line is given as x= the x-value of the point.

   Example: Find the line parallel to x=5 and goes through the point (-10,23).
   Our parallel line is x=-10 (-10 is the x-value of the point).

4) If you are asked to find the perpendicular line given a vertical line and a point, the perpendicular line is y= the y-value of the point.

   Example: Find the line perpendicular to x=5 and goes through the point (-10,23).
   The line perpendicular line is y=23 (23 is the y-value of the point).

Horizontal Lines

A horizontal line is given as y=k, where k is a real number. The slope for a horizontal line is zero. There are four main types of questions concerning finding horizontal lines.

1) If you are asked to find a horizontal line given slope of zero and a point, the horizontal line is y= y-value of the point.

   Example: Find the equation of the line given m= 0 and (3,2).
   Since slope is zero, we have a horizontal line. The equation of the line is y=2 (2 is the y-value of the point).

2) If you are given two points and the y-values for those two points are the same, then the equation of the line that goes through those points is horizontal. The line is written y= y-value from the points. If you use the slope formula, you would get a zero in the denominator, which means the slope is zero. If you get a zero in both the numerator and denominator, then the line is vertical.

   Example: Find the equation of the line that goes through the points (-3,-9) and (4,-9).
   Notice that the y-values are the same for both points. This means we have a horizontal line y=-9 (-9 is the y-values for both points).

3) If you are asked to find a line parallel given a horizontal line and a point, your parallel line is given as y= the y-value of the point.

   Example: Find the line parallel to y=5 and goes through the point (-10,23).
   Our parallel line is y=23 (23 is the y-value of the point).

4) If you are asked to find the perpendicular line given a horizontal line and a point, the perpendicular line is x= the x-value of the point.

   Example: Find the line perpendicular to y=5 and goes through the point (-10,23).
   The line perpendicular line is x=-10 (-10 is the x-value of the point) .

Helpful websites:

http://www.purplemath.com/modules/strtlneq.htm

http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut16_eqline.htm

http://www.themathpage.com/alg/slope-of-a-line.htm

http://coolmath.com/algebra/08-lines/index.html