Logarithmic Equations

$\log_b a = x$ is equivalent to $a = b^x$

Example: $\log_2 8 = 3$, because $8 = 2^3$

The default base for logarithms is 10

Example: $\log 100$ is the same as $\log_{10} 100$

The natural log is $\ln x$, which has base e

$e \approx 2.718281828\ldots$

$\ln x = \log_e x$

Properties of Logarithmic Equations

$\log_a 1 = 0$

$\log_a a = 1$

$\ln e = 1$

$\log_a a^x = x$

$\ln e^x = x$

$\log_a b^x = x \log_a b$

$a^{\log_a b} = b$

$\log xy = \log x + \log y$

$\log \frac{x}{y} = \log x - \log y$

$\log_x y = \frac{\log y}{\log x}$

If $\log x = \log y$, then $x = y$

For $\log_a x$, the domain is $x > 0$, otherwise the logarithm is undefined

Solving an Exponential or Logarithmic Equation

Solve the equation: $4^x = 87$

$\log 4^x = \log 87$

*Take the log on both sides

$x \log 4 = \log 87$

*Move the exponent (x) to the front of the log

$x = \frac{\log 87}{\log 4} \approx 3.221$

*Divide both sides by $\log 4$

Solve the equation: $\log_5 x = 6$

$x = 5^6$ *Rearrange the equation so that the log is not needed anymore

$x = 15625$

Expand the following expression and simplify: $\ln 15ab^3$

$\ln 15ab^3 = \ln 15 + \ln a + \ln b^3$ *Expand the expression

$\ln 15 + \ln a + \ln b^3 = \ln 15 + \ln a + 3\ln b$ *Move the exponent (3) to the front of the natural log

The final answer is $\ln 15 + \ln a + 3\ln b$

Compound Interest Questions

General Compound Interest Equation: $A = P(1 + \frac{r}{n})^{nt}$

Compounded Continuously: $A = Pe^{rt}$

A = Amount at the end of the time period

P = Principal amount put in

r = Interest rate (as a decimal)

n = Number of times compounded per year

t = Length of the time period in years

$e \approx 2.718281828 \ldots \rightarrow \mathit{Use\ the\ } e^x \mathit{\ button\ on\ your\ calculator}$

Examples: $1200 is put into a bank account that earns 8% interest. Find the amount of money in the bank account after 5 years if the interest is compounded annually, monthly, and continuously. Round to the nearest cent.

In this particular problem:

A = Amount in the account after the 5 years (also what we are trying to find)

P = $1200

r = 8% = .08

t = 5 years

n = 1 for annually

n = 12 for monthly

Annually: $A = P(1 + \frac{r}{n})^{nt}$

$A = 1200(1 + \frac{.08}{1})^{1*5}$

$A = 1200(1.08)^5$

$A = \$\,1763.19$

*Make sure the 1.08 is in parenthesis in your calculator so that the correct order of operations is followed

Monthly: $A = P(1 + \frac{r}{n})^{nt}$

$A = 1200(1 + \frac{.08}{12})^{12*5}$

$A = 1200(1.00\overline{6})^{60}$

$A = \$\,1787.81$

Continuously: $A = Pe^{rt}$

$A = 1200e^{.08*5}$

$A = 1200e^{.4}$

$A = \$\,1790.19$

You are trying to save $80,000 to help you purchase a home. You start with $25,000 in a bank account that gains 4% interest. How long will it take you to save the $80,000 you need if the interest is compounded annually, quarterly, and continuously? Round to the nearest hundredth.

In this particular problem:

A = $80,000

P = $25,000

r = 5% = .05

t = How long it will take you save the money you need (what we are trying to find)

n = 1 for annually

n = 4 for quarterly

Annually: $A = P(1 + \frac{r}{n})^{nt}$

$80000 = 25000(1 + \frac{.05}{1})^{1*t}$

$\frac{80000}{25000} = (1 + .05)^t$ *Divide by 25000 on both sides

$3.2 = (1.05)^t$ *Add inside the parenthesis

$\log 3.2 = \log 1.05^t$ *Take the log on both sides

$\log 3.2 = t \log 1.05$ *Move the exponent (t) to the front of the log

$\frac{\log 3.2}{\log 1.05} = t$ *Divide by log 1.05 on each side

$t = 23.84\ years$

Quarterly: $A = P(1 + \frac{r}{n})^{nt}$

$80000 = 25000(1 + \frac{.05}{4})^{4*t}$

$\frac{80000}{25000} = (1 + \frac{.05}{4})^{4t}$ *Divide by 25000 on both sides

$3.2 = (1.0125)^{4t}$ *Add inside the parenthesis

$\log 3.2 = \log 1.0125^{4t}$ *Take the log on both sides

$\log 3.2 = 4t \log 1.0125$ *Move the exponent (4t) to the front of the log

$\frac{\log 3.2}{\log 1.0125} = 4t$ *Divide by log 1.0125 on each side

$4t = 93.6324$

$t = 23.41\ years$

Continuously: $A = Pe^{rt}$

$80000 = 25000e^{.05*t}$

$\frac{80000}{25000} = e^{.05t}$ *Divide by 25000 on both sides

$3.2 = e^{.05t}$

$\ln 3.2 = \ln e^{.05t}$ *Take the natural log on both sides (ln goes with e)

$\ln 3.2 = .05t\ \ln e$ *Move the exponent (.05t) to the front of the log

$\ln 3.2 = .05t$ *$\ln e = 1$

$t = \frac{\ln 3.2}{.05}$ *Divide by .05 on both sides

$t = 23.26\ years$