Simplifying Square Roots

- Using the list of perfect squares, split up the square root into two separate square roots, one for each of the factors (one of them must be on the list of perfect squares). Put the perfect square one first.

  List of perfect squares:

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

  In this particular instance, 4 is a number that goes into 48 (4*12=48)

  $$\sqrt{48} = \sqrt{4} \cdot \sqrt{12}$$

- Take the square root of the one with the perfect square

  $$\sqrt{48} = 2\sqrt{12}$$

- Check the remaining square root to see if the process can be repeated
  - 12 can be split up as 4 and 3

  $$\sqrt{48} = 2 \cdot \sqrt{4} \cdot \sqrt{3}$$

  $$\sqrt{48} = 2 \cdot 2\sqrt{3}$$

  $$\sqrt{48} = 4\sqrt{3}$$

The process is exactly the same for roots higher than 2. Use the correct list of perfect cubes or higher to help split the original root.

x	**x^2**	**x^3**	**x^4**	**x^5**
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049
10	100	1000	10000	100000
11	121	1331	14641	161051
12	144	1728	20736	248832
13	169	2197	28561	371293
14	196	2744	38416	537824
15	225	3375	50625	759375
16	256	4096	65536	1048576
17	289	4913	83521	1419857
18	324	5832	104976	1889568
19	361	6859	130321	2476099
20	400	8000	160000	3200000

Examples: Simplify the following roots.

$\sqrt{80}$ $\sqrt[3]{243}$ $\sqrt{700}$ $\sqrt[5]{128}$ $\sqrt{575}$ $\sqrt[3]{792}$ $\sqrt{117}$

$$\sqrt{80} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$$

$$\sqrt[3]{243} = \sqrt[3]{27} \cdot \sqrt[3]{9} = 3\sqrt[3]{9}$$

$$\sqrt{700} = \sqrt{100} \cdot \sqrt{7} = 10\sqrt{7}$$

$$\sqrt[5]{128} = \sqrt[5]{32} \cdot \sqrt[5]{4} = 2\sqrt[5]{4}$$

$$\sqrt{575} = \sqrt{25} \cdot \sqrt{23} = 5\sqrt{23}$$

$$\sqrt[3]{792} = \sqrt[3]{8} \cdot \sqrt[3]{99} = 2\sqrt[3]{99}$$

$$\sqrt{117} = \sqrt{9} \cdot \sqrt{13} = 3\sqrt{13}$$

- When there are variables underneath the root
  - Take the exponent on each variable, and determine how many groups of the root can be made from that number

    $$\sqrt[3]{x^{17}}, \textit{there are } 5 \textit{ groups of } 3\ (\textit{the root}) \textit{ that can be made}$$

  - Put the variable on the outside, with the exponent being however many groups were determined. Underneath the square root will be any leftovers of that variable.

    $$\sqrt[3]{x^{17}} = \sqrt[3]{x^{15}} \cdot \sqrt[3]{x^2} = x^5\sqrt[3]{x^2}, \textit{because } 5 \textit{ groups of } 3 \textit{ is } 15. \textit{By taking}$$

    $$15\ x's \textit{ outfrom under the root, of the } 17\ x's \textit{ we started with, } 2 \textit{ will remain,}$$

    $$\textit{leaving us with } x^2 \textit{ underneath the root, and } x^5 \textit{ outside of the root}$$

Examples: Simplify the following roots.

$\sqrt{y^9}$ $\sqrt{x^4}$ $\sqrt[3]{a^8b^{14}}$ $\sqrt[3]{xy^3z^{10}}$ $\sqrt[4]{a^{15}b^9}$ $\sqrt{20xy^5}$ $\sqrt[3]{64a^{13}b^3c^8}$

$$\sqrt{y^9} = \sqrt{y^8} \cdot \sqrt{y} = y^4\sqrt{y}$$

$$\sqrt{x^4} = x^2$$

$$\sqrt[3]{a^8b^{14}} = \sqrt[3]{a^6b^{12}} \cdot \sqrt[3]{a^2b^2} = a^2b^4\sqrt[3]{a^2b^2}$$

$$\sqrt[3]{xy^3z^{10}} = \sqrt[3]{y^3z^9} \cdot \sqrt[3]{xz} = yz^3\sqrt[3]{xz}$$

$$\sqrt[4]{a^{15}b^9} = \sqrt[4]{a^{12}b^8} \cdot \sqrt[4]{a^3b} = a^3b^2\sqrt[4]{a^3b}$$

$$\sqrt{20xy^5} = \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{y^4} \cdot \sqrt{xy} = 2y^2\sqrt{5xy}$$

$$\sqrt[3]{64a^{13}b^3c^8} = \sqrt[3]{64} \cdot \sqrt[3]{a^{12}b^3c^6} \cdot \sqrt[3]{ac^2} = 4a^4bc^2\sqrt[3]{ac^2}$$