

# Stats Review

## Chapter 10

# Note:

This review is meant to highlight basic concepts from the course. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

The questions are displayed on one slide followed by the answers are displayed in **red** on the next.

This review is available in alternate formats upon request.

# Hypothesis Testing

Determine if it is a right-, left-, or two-tailed test

$$H_0: \mu = 6$$

$$H_1: \mu > 6$$

$$H_0: \mu = 6$$

$$H_1: \mu \neq 6$$

$$H_0: \mu = 6$$

$$H_1: \mu < 6$$

The mean age of lawyers in New York is 50.7 years.

The mean annual return for an employee's IRA is at most 3.4 percent.

A popular referendum on the ballot is favored by more than half of the voters.

# Hypothesis Testing

Determine if it is a right-, left-, or two-tailed test

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$$H_1: \mu > 6$$

right-tailed

$$H_0: \mu = 6$$

$$H_1: \mu \neq 6$$

two-tailed

$$H_0: \mu = 6$$

$$H_1: \mu < 6$$

left-tailed

The mean age of lawyers in New York is 50.7 years.

Two-tailed

The mean annual return for an employee's IRA is at most 3.4 percent.

Left-Tailed

A popular referendum on the ballot is favored by more than half of the voters.

Right-Tailed

## Type I and Type II errors

Determine the type I and type II error. Find the probability of making a type I error.

A referendum for an upcoming election is favored by less than half of the voters. Level of significance,  $\alpha$ , is 0.01.

According to the Project on Student Debt 2013, the average student debt for college students is more than \$30,000. Suppose the level of significance is Level of significance,  $\alpha$ , is 0.005.

## Type I and Type II errors

Determine the type I and type II error. Find the probability of making a type I error.

A referendum for an upcoming election is favored by less than half of the voters. Level of significance,  $\alpha$ , is 0.01.

Type I error: rejecting that  $p=.5$ , when in reality  $p<.5$ . You reject the null hypothesis that the referendum is favored by 50% (half), when it is the true percentage of favoring the referendum.

Type II error: not rejecting  $p=.5$ , when  $p<.5$ . You fail to reject the null hypothesis that the referendum is favored by half, when the true referendum percentage is less than 50%.

Probability(type I error)= $\alpha=.01$

According the Project on Student Debt 2013, the average student debt for college students is more than \$30,000. Suppose the level of significance is Level of significance,  $\alpha$ , is 0.005.

Type I error: rejecting that  $\mu=30000$ , when in reality  $\mu>30000$ . You reject the null hypothesis that the student debt is \$30,000, when it is the true student debt mean.

Type II error: not rejecting  $\mu=30000$ , , when  $\mu>30000$ . You fail to reject the null hypothesis that the student debt is \$30,000, when the true student debt is more than \$30,000.

Probability(type I error)= $\alpha=.005$

## Proper Conclusion for Hypothesis Test

In 2011, Logitech found that the of all lost remotes, 4% were found in the refrigerator. Another researcher believes that the percentage has since increased. What is a proper conclusion given that

a) The null hypothesis is rejected

b) the null hypothesis is not rejected

## Proper Conclusion for Hypothesis Test

In 2011, Logitech found that the of all lost remotes, 4% were found in the refrigerator. Another researcher believes that the percentage has since increased. What is a proper conclusion given that

a) The null hypothesis is rejected

There is sufficient evidence to conclude that the percentage of remotes found in the refrigerator is more than 4%.

b) the null hypothesis is not rejected

There is not sufficient evidence to conclude that the percentage of remotes found in refrigerators has increased.



## Hypothesis Test for Population Proportion: Classical Approach

A survey of 1000 adults, 542 found that they could not eat just one m&m. Does this sample evidence find that more than half of adults cannot eat just one m&m? Use the  $\alpha=.05$  level of significance.

# Hypothesis Test for Population Proportion: Classical Approach

A survey of 1000 adults, 542 found that they could not eat just one m&m. Does this sample evidence find that more than half of adults cannot eat just one m&m? Use the  $\alpha=.05$  level of significance.

All conditions met:

- From a randomized experiment
- $np_0(1 - p_0) = 1000(.5)(1 - .5) = 250 \geq 10$
- Independent samples

State the null and alternative Hypothesis

$$H_0: p = .5$$

$$H_1: p > .5$$

Find  $\hat{p}$ :

$$\hat{p} = \frac{x}{n} = \frac{542}{1000} = .542$$

Compute the test statistic,  $z_0$

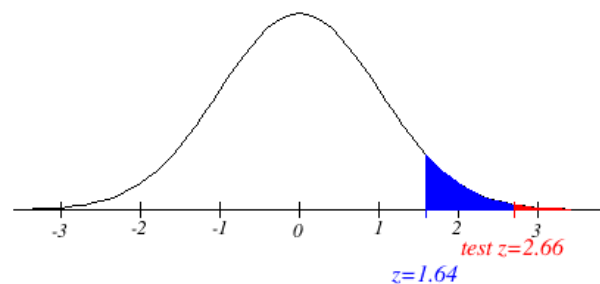
$$z_0 = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.542 - .5}{\sqrt{\frac{.5(1-.5)}{1000}}} = 2.66$$

Determine the critical value: For right tailed,  $z_\alpha = z_{.05} = 1.645$

Compare with the test statistic with the critical value: For a right-tailed, we reject the null hypothesis if  $z_0 > z_\alpha$

$2.66 > 1.645$  Therefore we reject the null hypothesis.

**Conclusion: We reject the null hypothesis. There is sufficient evidence at the  $\alpha=.05$  level of significance to conclude more than half of adults cannot eat just 1 m&m.**



## Hypothesis Test for Population Proportion: p-value Approach

A study found that 8 percent of internet users do not use email or search engines. A researcher thought thinks the percent is different. A random sample of 175 people were selected and asked if they use email or search engines. 11 do not use email or search engines. Test the researcher's claim at  $\alpha=.05$ .

## Hypothesis Test for Population Proportion: p-value Approach

A study found that 8 percent of internet users do not use email or search engines. A researcher thought thinks the percent is different. A random sample of 175 people were selected and asked if they use email or search engines. 11 do not use email or search engines. Test the researcher's claim at  $\alpha=.05$ .

All conditions met:

- From a randomized experiment
- $np_0(1 - p_0) = 175(.08)(1 - .08) = 12.88 \geq 10$
- Independent samples

State the null and alternative Hypothesis

$$H_0: p = .08$$

$$H_1: p \neq .08$$

Find  $\hat{p}$ :

$$\hat{p} = \frac{x}{n} = \frac{11}{175} = .06286$$

Compute the test statistic,  $z_0$

$$z_0 = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.06286 - .08}{\sqrt{\frac{.08(1-.08)}{175}}} = -.84$$

Determine the p-value

When  $z = -.84$ , the left area is .2005. Doubling this (two-tailed) we get .4010.

The p-value is greater than  $\alpha$ , so do not reject null hypothesis.

**Conclusion: We do not reject the null hypothesis. There is not sufficient evidence at the  $\alpha=.05$  level of significance to conclude internet users who not use e-mail or search engines is different than 8%.**

## Hypothesis Test for Population Proportion

In 2003, 4.8 % of males living in New England are retired fishermen. A researcher believes this number has increased since then. She randomly samples 50 male and finds that 6 are retired fisherman. Test this researcher's claim at the  $\alpha=.1$  level of significance.

## Hypothesis Test for Population Proportion

In 2003, 4.8 % of males living in New England are retired fishermen. A researcher believes this number has increased since then. She randomly samples 50 male and finds that 6 are retired fisherman. Test this researcher's claim at the  $\alpha=.1$  level of significance.

Check Conditions:

- From a randomized experiment
- $np_0(1 - p_0) = 50(.048)(1 - .048) = 2.2848 \geq 10$  This condition is NOT MET!
- Independent samples

Since  $np_0(1 - p_0) < 10$ , we need to use the binomial probability distribution.

State the null and alternative Hypothesis

$$H_0: p = .048$$

$$H_1: p > .048$$

$$P(x \geq 6) = 1 - P(x < 6) = 1 - P(x \leq 5)$$

Use  $P(x) = {}_n C_x p^x (1-p)^{n-x}$  for  $x=0,1,2,3,4,5$  or use technology where  $n=50$  and  $p=.048$

$$P(0) = {}_{50}C_0 (.048)^0 (1-.048)^{50-0} = .08548$$

$$P(1) = .21549, P(2) = .26619, P(3) = .21474, P(4) = .12722, P(5) = .05901$$

$$P(x \leq 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = .96813$$

$$P(x \geq 6) = 1 - P(x \leq 5) = 1 - .96813 = .03187$$

The p-value of .03187 is less than .1, so reject the null hypothesis.

**There is sufficient evidence at the  $\alpha=.1$  level to conclude that the percentage of males living in New England that are retired fishermen has increased.**

## Hypothesis Test for Population Mean: P-Value Approach

A local retailer claims that the mean waiting time is less than 9 minutes. A random sample of 40 waiting times has a mean of 8.4 minutes with a standard deviation of 2.1 minutes. At  $\alpha = 0.01$ , test the retailer's claim. Assume the distribution is normally distributed. Test the retailer's claim using  $\alpha = .01$ .

## Hypothesis Test for Population Mean: P-Value Approach

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1) State the null and alternative hypothesis

$$H_0: \mu = 9$$

$$H_1: \mu < 9$$

2) Find the t-value

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{8.4 - 9}{2.1 / \sqrt{40}} = -1.807 \text{ with 39 degrees of freedom}$$

3) Find the p-value

Actual P-value = .03923 (must use technology to get the actual p-value)

Approximate p-value = Between .025 and .05 (from table)

4) Conclusion: **Since the p-value is less than .01 ( $\alpha$ ), we reject the hypothesis. There is significant evidence that the mean waiting time is less than 9 minutes.**



## Hypothesis Test for Population Mean

In 2002, the mean age of inmate on death-row was 40.7 years. A researcher wonders if the mean age has changed since then. She randomly selects 32 death-row inmates and finds that their mean age is 38.9 with a standard deviation of 9.6. Test the researchers claim.  $\alpha=.05$

## Hypothesis Test for Population Mean

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1) State the Null and Alternative hypothesis

$$h_0: \mu = 40.7$$

$$h_1: \mu \neq 40.7$$

2) Calculate the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{38.9 - 40.7}{9.6/\sqrt{32}} = -1.06 \text{ with 31 degrees of freedom}$$

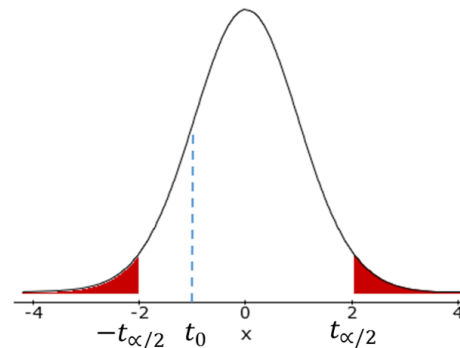
3) p-value approach: Find the p-value (use technology). If using the table, you need to multiply the p-value by 2

P-value = .2970

The p-value is greater than 5% ( $\alpha$ ), so we do not reject the null hypothesis.

Classical Approach: Using 31 degrees of freedom,  $t_{\alpha/2}$ , Critical value (from table is)  $\pm 2.040$

Since the test statistic is not in the rejection region, we do not reject the null hypothesis.



**Conclusion: There is not sufficient evidence to reject the claim that the mean has changed.**

## Hypothesis Test for Population Standard Deviation

A professor at an all-men's college determined that the standard deviation of men's heights is 2.5 inches. The professor then randomly selected 41 female students from a nearby all-female college and found the standard deviation to be 2.9 inches. Test the professor's claim that the standard deviation of female heights is greater than 2.5 inches. Use  $\alpha = 0.01$  and the p-value approach.

## Hypothesis Test for Population Standard Deviation

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1. State the Null and Alternative hypothesis

$$h_0: \sigma = 2.5$$

$$h_1: \sigma > 2.5$$

2. Calculate the test statistic

$$\chi_0^2 = \frac{(n - 1)s^2}{\sigma_0^2} = \frac{(41 - 1)2.9^2}{2.5^2} = 53.824$$

3. P-value Approach Determine the P-value

From Technology: p-value=.0709

From Table: With 40 degrees of freedom, 53.824 has a p-value between .05 and .1

4. Conclusion: p-value  $>$   $\alpha$  (.01)

**Fail to reject the null hypothesis. There is not statistical evidence to support the claim that standard deviation is greater than 2.5.**

## When to Use t and When to Use z

- Use Z
  - Inference (confidence interval/ hypothesis testing) about population proportion
  - Looking for sample sizes
  - Have population standard deviation  $\sigma$
- Use T
  - Inference (confidence interval/ hypothesis testing) about population mean
  - Sample standard deviation  $s$  (but not looking for sample size  $n$ )